Sydney Technical High School

Student Name ______________________________

Teacher's Name: ______________________________

Extension 1 Mathematics

TRIAL HSC

August 2021

General Instructions

- Working time - 120 minutes + 10 minutes reading time
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt questions 11-14
- Allow about 1 hours and 45 minutes for this section

Section I

10 Marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1. Which of the following expressions is a correct factorisation of $x^3 - 27$?

(A) $(x-3)\,(x^2 - 3x + 9)$

(B) $(x-3)\,(x^2 - 6x + 9)$

(C) $(x-3)\,(x^2 + 3x + 9)$

(D) $(x-3)\,(x^2 + 6x + 9)$

2. What is the domain of $y = \cos^{-1}\left(\frac{3x}{2}\right)$?

(A) $x \in [-\frac{2}{3}, \frac{2}{3}]$

(B) $x \in (-\frac{2}{3}, \frac{2}{3})$

(C) $x \in [-\frac{3}{2}, \frac{3}{2}]$

(D) $x \in (-\frac{3}{2}, \frac{3}{2})$

3. A coin is biased such that the probability of a head is 0.7. The probability that exactly three tails will be observed when the coin is flipped eight times is:

(A) $8 \times 0.3^5 \times 0.7^3$

(B) ${}^8\mathbf{C}_3 \times 0.3^5 \times 0.7^3$

(C) ${}^8\mathbf{C}_3 \times 0.3^3 \times 0.7^5$

(D) $8 \times 0.3^3 \times 0.7^5$

4. Which one of the following vectors is parallel to the vector $\overrightarrow{OP} = -2\underset{\sim}{i} + \underset{\sim}{j}$?

(A) $\overrightarrow{OA} = -12\underset{\sim}{i} + 8\underset{\sim}{j}$

(B) $\overrightarrow{OB} = -\underset{\sim}{i} + 2\underset{\sim}{j}$

(C) $\overrightarrow{OC} = 2\underset{\sim}{i} + \underset{\sim}{j}$

(D) $\overrightarrow{OD} = 12\underset{\sim}{i} - 6\underset{\sim}{j}$

5. Which of the following is the correct expression for $\int \frac{dx}{\sqrt{16 - x^2}}$?

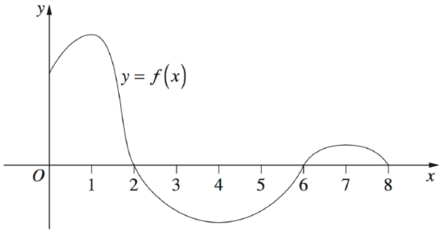
(A) $\sin^{-1} 4x + c$

(B) $\cos^{-1} 4x + c$

(C) $\sin^{-1} \frac{x}{4} + c$

(D) $\cos^{-1} \frac{x}{4} + c$

6. The graph of $y = f(x)$ has been drawn to scale for $0 \leq x \leq 8$.



Which of the following integrals has the greatest value?

(A) $\int_0^1 f(x)\, dx$

(B) $\int_0^2 f(x)\, dx$

(C) $\int_0^7 f(x)\, dx$

(D) $\int_0^8 f(x)\, dx$

7. Given that $|\underset{\sim}{u}| = |\underset{\sim}{v}| = 3$ and the angle between them is $150°$.

What is the value of $\underset{\sim}{u} \cdot \underset{\sim}{v}$?

(A) 9

(B) -4.5

(C) $-\frac{9\sqrt{3}}{2}$

(D) $8\sqrt{3}$

8. Which polynomial has a multiple zero at $x = 2$

(A) $x^3 + 3x^2 - 4$

(B) $x^3 - 3x + 2$

(C) $x^3 - 5x^2 + 8x - 4$

(D) $x^3 + 3x^2 + x - 2$

9. When $\sqrt{3}sinx - cosx$ is rewritten in the form $R\sin(x-\alpha)$, then:

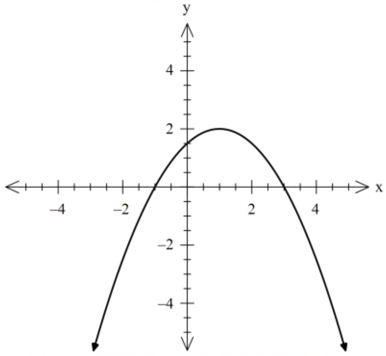
(A) $R = 2$ and $\alpha = \frac{\pi}{6}$

(B) $R = 2$ and $\alpha = \frac{5\pi}{6}$

(C) $R = \sqrt{2}$ and $\alpha = \frac{\pi}{6}$

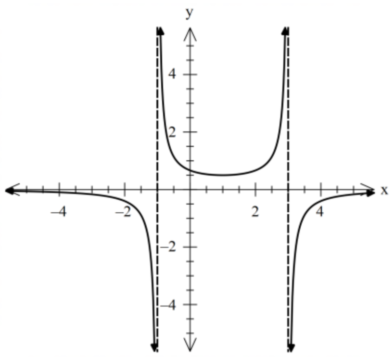
(D) $R = \sqrt{2}$ and $\alpha = \frac{5\pi}{6}$

10. The graph of the function $y = f(x)$ is below.

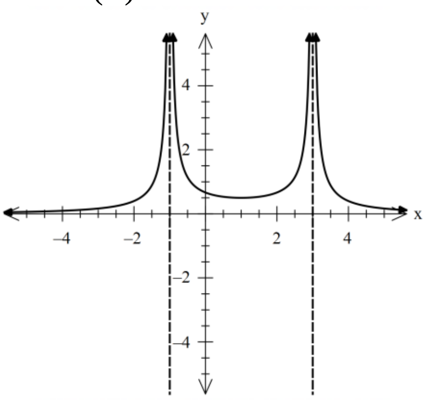


Which of the following is a graph of $y = -\frac{1}{|f(x)|}$?

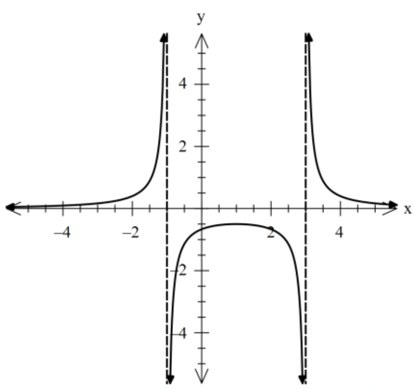
(A)



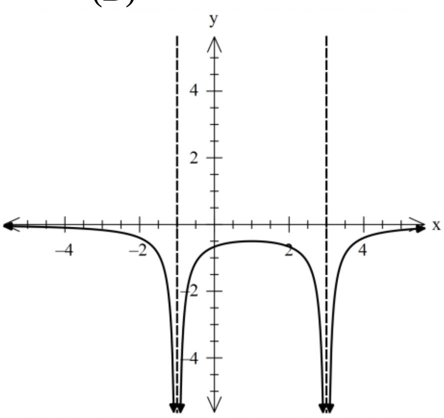
(B)



(C)



(D)



Section II

Total marks – 60
Attempt Question 11-14
Allow about 1 hour and 45 minutes for this section

<u>Begin each question on a NEW page</u>

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Begin a NEW page.

a) Find the equation of the line which is parallel to $2x + y - 3 = 0$ and passes through (2, 1). **2**

Leave your answer in general form.

b) Solve $|2x - 1| > 4$ **2**

c) 2 cards are chosen at random, without replacement, from a standard 52-deck of playing cards. Find the probability of choosing at least 1 heart. **2**

d) Solve the inequality $\frac{1-x}{1+x} \leq 1$ **3**

e) Peter and May are two of ten candidates for a committee. **2**

How many ways can a committee of five be chosen, if Peter refuses to be on the same committee as May?

f) The weights in a population are normally distributed with a mean of 80 kg and a standard deviation of 8 kg. Use the empirical rule to find the approximate probability that a randomly selected person has a weight under 64 kg? **2**

g) Find $\frac{dy}{dx}$ if $y = e^{-x} \sin^{-1} x$ **2**

End of Question 11

Question 12 (15 marks) Begin a NEW page.

a) i) Using t-results, prove that $\cot\left(\frac{\theta}{2}\right) = \frac{sin\theta}{1-cos\theta}$. **2**

ii) Hence find the exact value of $\cot\left(\frac{\theta}{2}\right)$ given that $sin\theta = \frac{5}{6}$ and $\frac{\pi}{2} < \theta < \pi$. **2**
You don't need to rationalise the denominator of your answer.

b) Use the substitution $u = 2e^x$ to show that **3**

$$\int_{\ln\left(\frac{1}{2}\right)}^{\ln\left(\frac{\sqrt{3}}{2}\right)} \frac{e^x}{1+4e^{2x}}\, dx = \frac{\pi}{24}$$

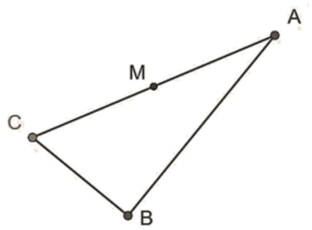
c) i) Records show that 64% of students at a school travelled to and from school by bus. **2**

Samples of 100 students at the school are taken to determine the proportion who travel to and from school by bus. Show that the distribution of such sample proportions has mean 0.64 and standard deviation 0.048.

ii) Use the table below of $P(Z < z)$, where Z has a standard normal distribution, to estimate the probability that a sample of 100 students will contain at least 58 and at most 64 students who travel to and from school by bus. **2**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

d) ΔABC is a right-angled triangle with M being the midpoint of the hypotenuse AC, as shown below. Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



i) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

ii) Prove that M is equidistant from the three vertices of ΔABC **2**

End of Question 12

Question 13 (15 marks) Begin a NEW page.

a) A salad, which is initially at a temperature of $25°C$, is placed in a refrigerator that has a constant temperature of $3°C$. The cooling rate of the salad is proportional to the difference between the temperature in the refrigerator and the temperature, T, of the salad. That is T satisfies the equation

$$\frac{dT}{dt} = -k(T-3)$$

where t is the number of minutes after the salad is placed in the refrigerator.

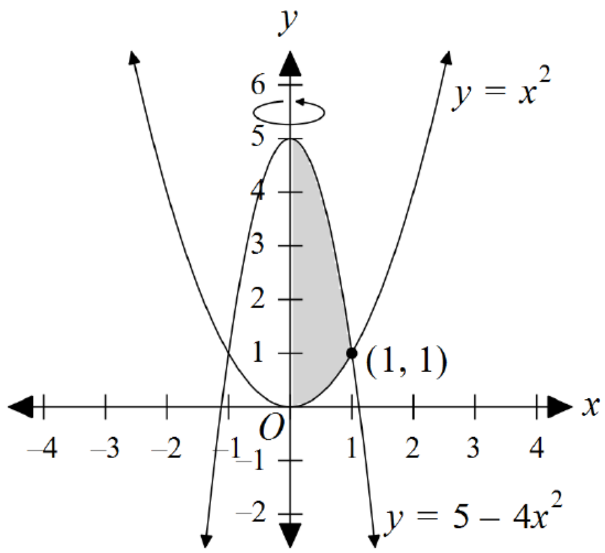
i) Show that $T = 3 + Ae^{-kt}$ satisfies this equation. **2**

ii) The temperature of the salad is $11°C$ after 10 minutes. **3**

Find the temperature of the salad after 15 minutes, correct to 1 decimal place.

b) Let $P(x) = x^3 + ax^2 + bx + 5$ where a and b are real numbers. **3**
Find the values of a and b given that $(x-1)^2$ is a factor of $P(x)$.

c) A solid of revolution is to be formed by rotating the shaded area shown between the graphs y= x^2 and y= $5-4x^2$ about the **y-axis**.



Find the exact volume of the solid formed. **3**

d) Use mathematical induction to prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7 for integers $n \geq 1$. **4**

End of Question 13

Question 14 (15 marks) Begin a NEW page.

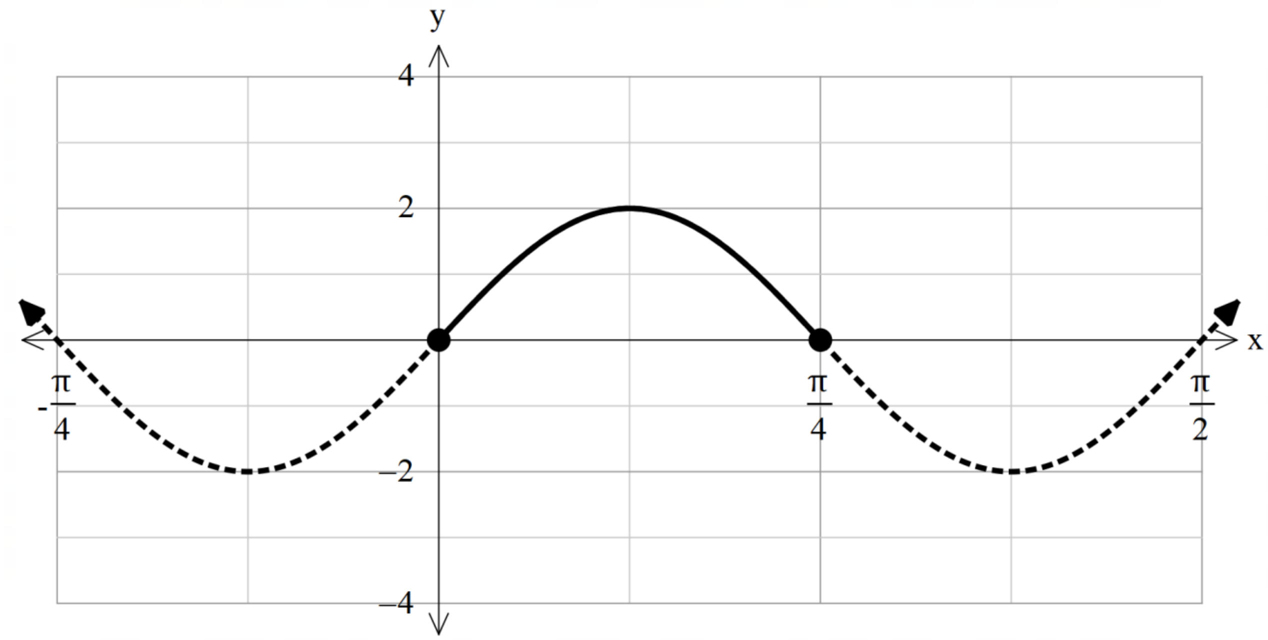
a) On a roulette wheel, there are 18 red numbers, 18 black numbers, and 1 green numbers. A ball is dropped onto the spinning wheel and lands on one of the numbers randomly.

Each result is independent. A gambler bets that the ball land on any of the black numbers.

i. If the gambler makes the same bet five times, let the random variable Y be the number of times the gambler wins. Write the distribution of Y in standard form, and give its mean and variance. **2**

ii. If the gambler makes the same bet five times, what is the probability he will win more times than he loses? Give your answer correct to three decimal places. **2**

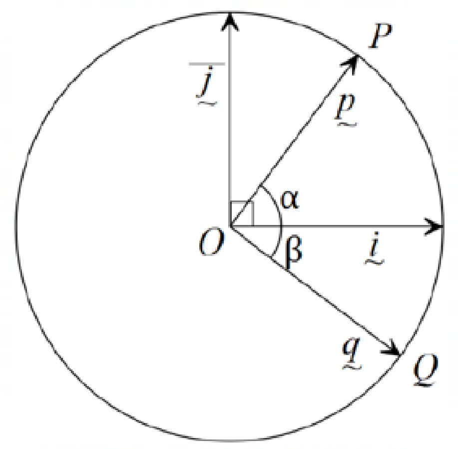
b) Consider $f(x) = 4\sin^2\left[2\left(x+\frac{\pi}{8}\right)\right] - 2, 0 < x < \frac{\pi}{4}$ and its graph below



i. Verify that $f(x)$ could be a probability density function. **3**
Hint: you must refer to both properties of a PDF.

ii. Find the mode of such a distribution. **1**

c) Use the unit circle and the vectors in the diagram to derive the expansion
$\cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta$ **3**



d) Find the six solutions of the equation: **4**

$$\sin\left(2\cos^{-1}(\cot(2\tan^{-1}x))\right) = 0$$

Give your answers as simplified surds.

End of Exam

Multiple Choice Answers

1. C
2. A
3. C
4. D
5. C
6. B
7. C
8. C
9. A
10. D

Q 11 (a)

$$2x + y - 3 = 0$$

$$y = -2x + 3$$

$$m = -2$$

$$y - 1 = -2(x - 2)$$

$$y - 1 = -2x + 4$$

$$2x + y - 5 = 0$$

Q 11 (c) $1 - P(\text{No heart})$

$$= 1 - \frac{3}{4} \times \frac{38}{51}$$

$$= \frac{15}{34}$$

Q 11 (b) $|2x - 1| > 4$

$2x - 1 > 4$ or $2x - 1 < -4$

$2x > 5$ or $2x < -3$

$x > \frac{5}{2}$ or $x < -\frac{3}{2}$

Q11(d) $\frac{1-x}{1+x} \leq 1$

$1+x \neq 0$

$x \neq -1$

$1-x = 1+x$

$2x = 0$

$x = 0$

$x < -1 \;, x \geq 0$

Q11(e) ${}^{10}C_5 - {}^{8}C_5 = 196$

${}^{10}C_5 - {}^{8}C_3$

Q11(f)

64 kg is 2 standard deviation less than 80kg

So the answer is 0.025

Q11(g) $\frac{dy}{dx} = -e^{-x}\sin^{-1}x + e^{-x}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$= -e^{-x}\sin^{-1}x + \frac{e^{-x}}{\sqrt{1-x^2}}$

$= -e^{-x}\sin^{-1}x + \frac{1}{e^{x}\sqrt{1-x^2}}$

Q 12 (a) (i) Let $t = \tan\frac{\theta}{2}$

$$RHS = \frac{\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}}$$

$$= \frac{2t}{1+t^2-(1-t^2)}$$

$$= \frac{2t}{2t^2}$$

$$= \frac{1}{t} = \cot\left(\frac{\theta}{2}\right) = LHS$$

12(a)(ii)

$$\frac{\pi}{2} < \theta \leq \pi$$

$$\sin\theta = \frac{5}{6} \qquad \cos\theta = -\frac{\sqrt{11}}{6}$$

$$\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$$

$$\cot\left(\frac{\theta}{2}\right) = \frac{\frac{5}{6}}{1-\left(-\frac{\sqrt{11}}{6}\right)}$$

$$= \frac{\frac{5}{6}}{\frac{6+\sqrt{11}}{6}}$$

$$= \frac{5}{6+\sqrt{11}}$$

12(b) $I = \int_{\ln\frac{1}{2}}^{\ln\frac{\sqrt{3}}{2}} \frac{e^x}{1+4e^{2x}}\,dx$

Let $u = 2e^x$

$$du = 2e^x\,dx$$

$$x = \ln\frac{\sqrt{3}}{2}, \quad u = \sqrt{3}$$

$$x = \ln\frac{1}{2}, \quad u = 1$$

$$I = \frac{1}{2}\int_{\ln\frac{1}{2}}^{\ln\frac{\sqrt{3}}{2}} \frac{2e^x}{1+(2e^x)^2}\,dx$$

$$= \frac{1}{2}\int_1^{\sqrt{3}} \frac{du}{1+u^2}$$

$$= \frac{1}{2}\left[\tan^{-1}u\right]_1^{\sqrt{3}}$$

$$= \frac{1}{2}\left[\tan^{-1}\sqrt{3} - \tan^{-1}1\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{24}$$

12(c)(i) For students in the sample, the number of students traveling to and from school by bus is a random variable X with the Binomial distribution $B(100, 0.64)$. Hence the sample proportion $\frac{X}{100}$ has mean $\mu = 0.64$

since $E\left(\frac{X}{n}\right) = \frac{np}{n} = p$

and standard deviation $\sigma = \sqrt{\frac{0.64 \times (1-0.64)}{100}}$

$= 0.048$

12 (c) (ii)

$P\left(0.58 \leq \frac{X}{100} \leq 0.64\right)$

Z scores for 0.64 $= \frac{0.64 - 0.64}{0.048} = 0$

Z scores for 0.58 $= \frac{0.58 - 0.64}{0.048} = -1.25$

$P(-1.25 \leq Z \leq 0)$

$= P(Z \leq 1.25) - 0.5$

$= 0.8944 - 0.5$

$= 0.3944$

12 (d) (i) $\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB}$

$= \overrightarrow{AM} - \overrightarrow{BM}$

$= \underset{\sim}{a} - \underset{\sim}{b}$

$\overrightarrow{BC} = \overrightarrow{BM} + \overrightarrow{MC}$

$= \overrightarrow{BM} + \overrightarrow{AM}$

$= \underset{\sim}{a} + \underset{\sim}{b}$

12 (d) (ii)

$\overrightarrow{AB} \cdot \overrightarrow{BC} = (\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b})$

$= \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{b} \cdot \underset{\sim}{b}$

$= |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2$

Since $\triangle ABC$ is a right-angled triangle

and $\overrightarrow{AB} \perp \overrightarrow{BC}$

then $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$

$|a|^2 - |b|^2 = 0$

$|a| = |b|$

So $|\overrightarrow{AM}| = |\overrightarrow{BM}| = |\overrightarrow{MC}|$

13(a) (i)

$$T = 3 + Ae^{-kt}$$

$$\frac{dT}{dt} = -kAe^{-kt}$$

$$= -k(T-3)$$

So $T = 3 + 4e^{-kt}$ satisfies the equation

13(a)(ii) $T = 3 + Ae^{-kt}$

At $t=0$, $T = 25$

$$25 = 3 + Ae^{0}$$

$$A = 22$$

$$T = 3 + 22e^{-kt}$$

When $t = 10$, $T = 11$

$$11 = 3 + 22e^{-10k}$$

$$8 = 22e^{-10t}$$

$$e^{-10k} = \frac{8}{22}$$

$$\ln e^{-10k} = \ln\frac{4}{11}$$

$$k = \frac{-1}{10}\left(\ln\frac{4}{11}\right)$$

When $t = 15$, $T = 3 + 22e^{-15\left(-\frac{1}{10}\ln\frac{4}{11}\right)}$

$$= 7.8241\ldots$$

$$\doteqdot 7.8 \text{ (to 1 decimal place)}$$

13(b) $P(x) = x^3 + ax^2 + bx + 5$

$P(1) = 1 + a + b + 5 = 0$

$a + b = -6 \quad \ldots \text{①}$

$P'(x) = 3x^2 + 2ax + b$

$P'(1) = 3 + 2a + b = 0$

$2a + b = -3 \quad \ldots \text{②}$

② − ① : $a = 3$

$b = -9$

13(c) $y = x^2$

$x = \sqrt{y}$

$y = 5 - 4x^2$

$x = \frac{\sqrt{5-y}}{2}$

$V = \int_0^1 \pi (\sqrt{y})^2 + \int_1^5 \pi \left(\frac{\sqrt{5-y}}{2}\right)^2 dy$

$= \pi \left(\int_0^1 y\,dy + \frac{1}{4} \int_1^5 (5-y)\,dy \right)$

$= \pi \left(\left[\frac{y^2}{2}\right]_0^1 + \frac{1}{4} \left[5y - \frac{y^2}{2}\right]_1^5 \right)$

$= \pi \left(\frac{1}{2} + \frac{1}{4} \left(5 \times 5 - \frac{25}{2} - 5 + \frac{1}{2}\right)\right)$

$= \pi \left(\frac{1}{2} + \frac{1}{4} (8) \right)$

$= \frac{5\pi}{2}$

13(d) Show true for $n=1$

$3^3 + 2^3 = 35$ which is divisible by 7.

True for $n=1$

Assume true for $n=k$

$3^{2k+1} + 2^{k+2} = 7p$ where p is an integer.

$3^{2k+1} = 7p - 2^{k+2}$

Prove that it is true for $n=k+1$

$LHS = 3^{2k+3} + 2^{k+3}$

$= 3^2 \cdot 3^{2k+1} + 2^{k+3}$

$= 3^2 (7p - 2^{k+2}) + 2^{k+3}$

$= 9 \times 7p - 9 \times 2^{k+2} + 2 \times 2^{k+2}$

$= 9 \times 7p - 7 \times 2^{k+2}$

$= 7(9p - 2^{k+2})$

$= RHS$

So It is true for $n=k$ then also true for $n=k+1$

By the principle of Mathematical induction, the result is true for all $n \geq 1$

14(a)(i)

This is a binomial random variable with $p = \frac{18}{37}$ and $n = 5$.

$Y \sim Bin(5, \frac{18}{37})$

$\mu = 5 \times \frac{18}{37} = \frac{90}{37} \approx 2.43$

$s^2 = 5 \times \frac{18}{37} \times \frac{19}{37} = \frac{1710}{1369} \approx 1.25$

14(a)(ii)

This is a binomial probability and we are looking for $P(w \geq 3)$ where w is the number of wins in 5 bets

$P(w \geq 3) = P(w=3) + P(w=4) + P(w=5)$

$= \binom{5}{3} \left(\frac{18}{37}\right)^3 \left(\frac{19}{37}\right)^2 + \binom{5}{4} \left(\frac{18}{37}\right)^4 \left(\frac{19}{37}\right)^1 + \binom{5}{5} \left(\frac{18}{37}\right)^5 \left(\frac{19}{37}\right)^0$

$= 0.475$

~~Factorisation~~

14 (b)(i) From Reference Sheet

$\sin^2 2(x+\frac{\pi}{8}) = \frac{1}{2}(1-\cos 4(x+\frac{\pi}{8}))$

For probability density function need to show $\int_a^b f(x)\,dx = 1$

and $f(x) \geq$ for $a \leq x \leq b$

$$\int_0^{\frac{\pi}{4}} (4\sin^2[2(x+\frac{\pi}{8})]-2)dx$$

$$= \int_0^{\frac{\pi}{4}} (\frac{4}{2}(1-\cos 4(x+\frac{\pi}{8})]-2)\,dx$$

$$= \int_0^{\frac{\pi}{4}} -2\cos 4(x+\frac{\pi}{8})\,dx$$

$$= [-\frac{1}{2}\sin 4(x+\frac{\pi}{8}]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{2}\sin(\pi+\frac{\pi}{2}) + \frac{1}{2}\sin\frac{\pi}{2}$$

$$= -1 \times -1 + \frac{1}{2}\times 1$$

$$= 1$$

For $0 \leq x \leq \frac{\pi}{4}$,

$$-\frac{1}{\sqrt{2}} \leq \sin 2(x+\frac{\pi}{8}) \leq 1$$

$$\frac{1}{2} \leq \sin^2 2(x+\frac{\pi}{8}) \leq 1$$

$$2 \leq 4\sin^2 2(x+\frac{\pi}{8}) \leq 4$$

$$0 \leq 4\sin^2 2(x+\frac{\pi}{8}) - 2 \leq 2$$

$\therefore f(x) \geq 0$ for all x in $0 \leq x \leq \frac{\pi}{4}$

(b)(ii) Mode = x value to give max $f(x)$

$$f(x) = 4\sin^2[2(x+\frac{\pi}{8})]-2$$

$$= -2\cos 4(x+\frac{\pi}{8}) \leftarrow \text{Max value} = 2$$

$$\cos 4(x+\frac{\pi}{8}) = -1$$

$$4(x+\frac{\pi}{8}) = \pi$$

$$x = \frac{\pi}{8}$$

14(c) $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}|\,|\underset{\sim}{v}| \cos\theta$

$$\cos\theta = \frac{u \cdot v}{|\underset{\sim}{u}|\,|\underset{\sim}{v}|}$$

Apply this to $\underset{\sim}{p}$ and $\underset{\sim}{q}$ which have length 1

$$\cos(\alpha+\beta) = \underset{\sim}{p} \cdot \underset{\sim}{q}$$ ✓

In order to find $\underset{\sim}{p} \cdot \underset{\sim}{q}$ we will find their components

$$\underset{\sim}{p} = \cos\alpha\, \underset{\sim}{i} + \sin\alpha\, \underset{\sim}{j}$$

$$\underset{\sim}{q} = \cos(-\beta)\, \underset{\sim}{i} + \sin(-\beta)\, \underset{\sim}{j}$$ ✓

$$= \cos\beta\, \underset{\sim}{i} - \sin\beta\, \underset{\sim}{j}$$

(vector $\underset{\sim}{q}$ is at $-\beta$ on the unit circle)

$$\cdot\, \underset{\sim}{q} = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$ ✓

So $\cos \alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

14(d) Let $\alpha = \tan^{-1} x$

$x = \tan\alpha$

Now $\cot(2\tan^{-1}x) = \frac{1}{\tan(2\tan^{-1}x)} = \frac{1}{\tan 2\alpha}$

$\cot 2\alpha = \frac{1}{\tan 2\alpha} = \frac{1-\tan^2\alpha}{2\tan\alpha}$

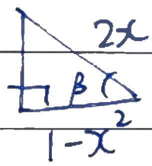
So. $\cot 2\alpha = \frac{1-x^2}{2x}$

Hence $\sin\left(2\cos^{-1}\left(\frac{1-x^2}{2x}\right)\right) = 0$

Let $\beta = \cos^{-1}\left(\frac{1-x^2}{2x}\right)$

then $\cos\beta = \frac{1-x^2}{2x}$ and $\sin 2\beta = 0$

Now $\sin 2\beta = 2\sin\beta\cos\beta$



So the third side

$= \sqrt{(2x)^2-(1-x^2)^2}$

$= \sqrt{4x^2-(1-2x^2+x^4)}$

$= \sqrt{-x^4+6x^2-1}$

So $\sin\beta = \frac{\sqrt{-x^4+6x^2-1}}{2x}$

Then $\sin 2\beta = 2\sin\beta\cos\beta = 2 \times \frac{\sqrt{-x^4+6x^2-1}}{2x} \times \frac{1-x^2}{2x} = 0$

which makes $1-x^2=0$ OR $-x^4+6x^2-1=0$

$x = \pm 1$

$x^4-6x^2+1=0$

$x^4-6x^2+9=8$

$(x^2-3) = 8$

$x = \pm(1\pm\sqrt{2})$

Solutions are: ± 1, $\pm 1+\sqrt{2}$, $\pm 1-\sqrt{2}$

$(OR \pm\sqrt{3+2\sqrt{2}})$ $(OR \pm\sqrt{3-2\sqrt{2}})$